Sample Paper 4

Leaving Certificate

Mathematics

Paper 1

Higher Level

Time: 2 hours, 30 minutes

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) If $z = -3 - i$, where $i = \sqrt{-1}$, find:

(i) $\bar{z}$,

(ii) the quadratic equation with roots z and $\bar{z}$.

(b) z is a root of the cubic equation $az^3 + 22z^2 + bz + 40 = 0$, where $a, b \in \mathbb{R}$.
Show that $a = 3$ and find b.
Write down all of the roots of the cubic equation.

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Question 2 **(25 marks)**

(a) If $\dfrac{ax}{b-c}=\dfrac{by}{c-a}=\dfrac{cz}{a-b}$, show $ax + by + cz = 0$.

(b) **(i)** Solve $x+\sqrt{5x+19}=-1$.

(ii) Solve $\log_2 x+\dfrac{12}{\log_2 x}=7,\ x>0,\ x\in\mathbb{R}$.

Question 3 **(25 marks)**

(a) Solve the simultaneous equations:

$$2x - y + 3z = 20$$
$$7x + y + z = 23$$
$$3x + y - z = 3$$

(b) Find the range of values of k for which $(2-3k)x^2 + (4-k)x + 2 = 0$ has no real roots.

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Question 4 **(25 marks)**

(a) Find the sum to infinity of the geometric sequence $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

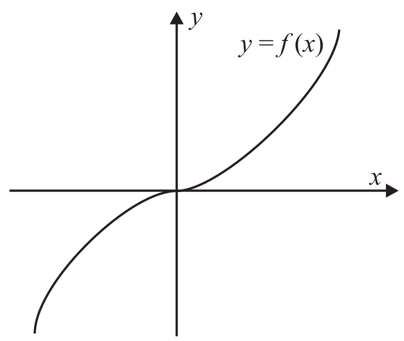
(b) The sum of three consecutive numbers in an arithmetic sequence is 27 and the sum of their squares is 293. Find the numbers.

Question 5 **(25 marks)**

Examine each of the situations below. Tick the box if you think the statement is correct or put an 'X' in the box if you think it is incorrect. Give reasons for your choices.

(a) $y = f(x)$ from $\mathbb{R}$ to $\mathbb{R}$.

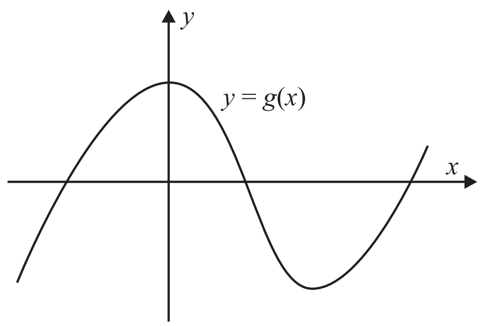
INJECTIVE FUNCTION ☐ SURJECTIVE FUNCTION ☐ BIJECTIVE FUNCTION ☐



REASON

(b) $y = g(x)$ from $\mathbb{R}$ to $\mathbb{R}$.

INJECTIVE FUNCTION ☐ SURJECTIVE FUNCTION ☐ BIJECTIVE FUNCTION ☐



REASON

(c) $y = x^2$ from $\mathbb{Z}^+ \cup \{0\}$ to the set of positive integers $\mathbb{Z}^+ \cup \{0\}$.

INJECTIVE FUNCTION ☐

SURJECTIVE FUNCTION ☐

BIJECTIVE FUNCTION ☐

REASON

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(d) $y = x^2$ from $\mathbb{R}^+$ to $\mathbb{R}^+$.

INJECTIVE FUNCTION

SURJECTIVE FUNCTION

BIJECTIVE FUNCTION

REASON

(e) $y = x^2$ from $\mathbb{R}$ to $\mathbb{R}^+$.

INJECTIVE FUNCTION

SURJECTIVE FUNCTION

BIJECTIVE FUNCTION

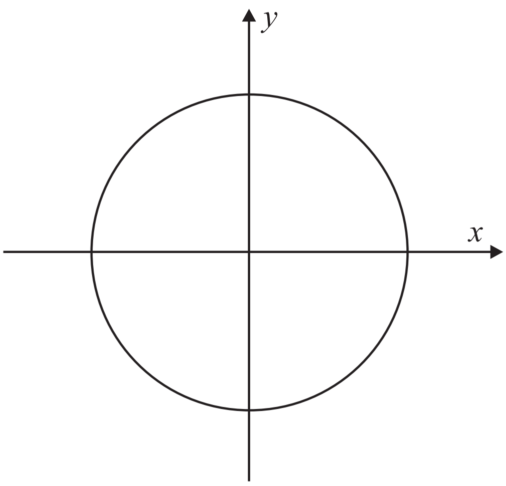
REASON

(f) $y = \pm\sqrt{1 - x^2}$.

INJECTIVE FUNCTION

SURJECTIVE FUNCTION

BIJECTIVE FUNCTION

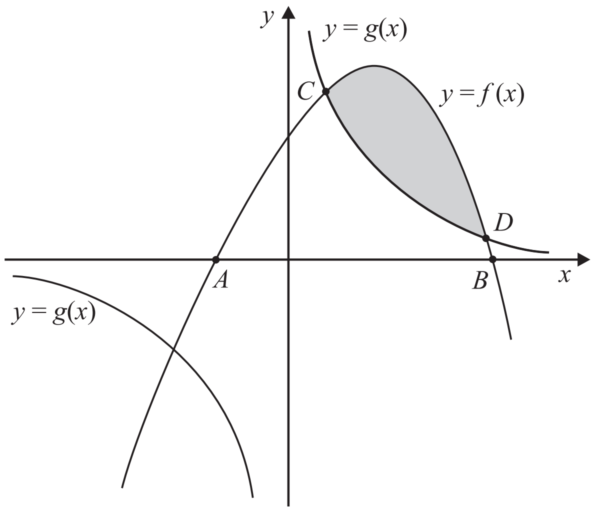


REASON

Question 6 **(25 marks)**

The graphs of $y = f(x) = -\frac{7}{6}x^2 + \frac{17}{6}x + \frac{31}{3}$ and $y = g(x) = \dfrac{12}{x}$ are shown below.

(a) Which curve is continuous? Why?



(b) Find the x-coordinate of the local maximum of $y = f(x)$.

(c) Find the coordinates of A and B, where $f(x)$ crosses the x-axis.

(d) Show that points $C(1, 12)$ and $D(4, 3)$ are on both curves.

(e) Find the shaded area between the two curves.

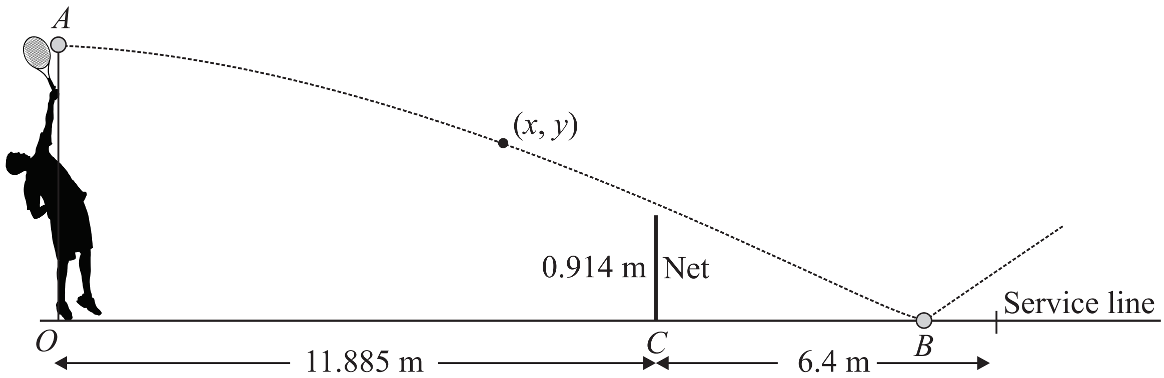
Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section.

Question 7 **(50 marks)**

A professional tennis player can serve the ball at 70 m/s. The ball leaves his racket from point A and travels along the trajectory AB. The x and y co-ordinates of the ball at any time t in seconds is given by:

$$x = 70t$$
$$y = 3.30 - 13.0t - 4.9t^2$$



(a) If the height of the player serving the ball is 1.85 m and his arm span is 0.8 m above his head, find the height above the base of the racket that the ball leaves the face of the racket.

(b) Find the time it takes, to two decimal places, for the ball to arrive at B.

(c) What is the distance $|OB|$? Is the serve in or out?

(d) Find the time it takes, to two decimal places, for the ball to get to C.

(e) What is the height of the ball, to three decimal places, above C? Does the ball clear the net?

(f) **(i)** If the ball just cleared the net, find the time, to three decimal places, it takes the ball to travel from the racket face to the net, assuming the same y equation.

(ii) As a result of the ball's new trajectory, the x equation has changed. Find b, to one decimal place, if $x = bt$.

Question 8 **(50 marks)**

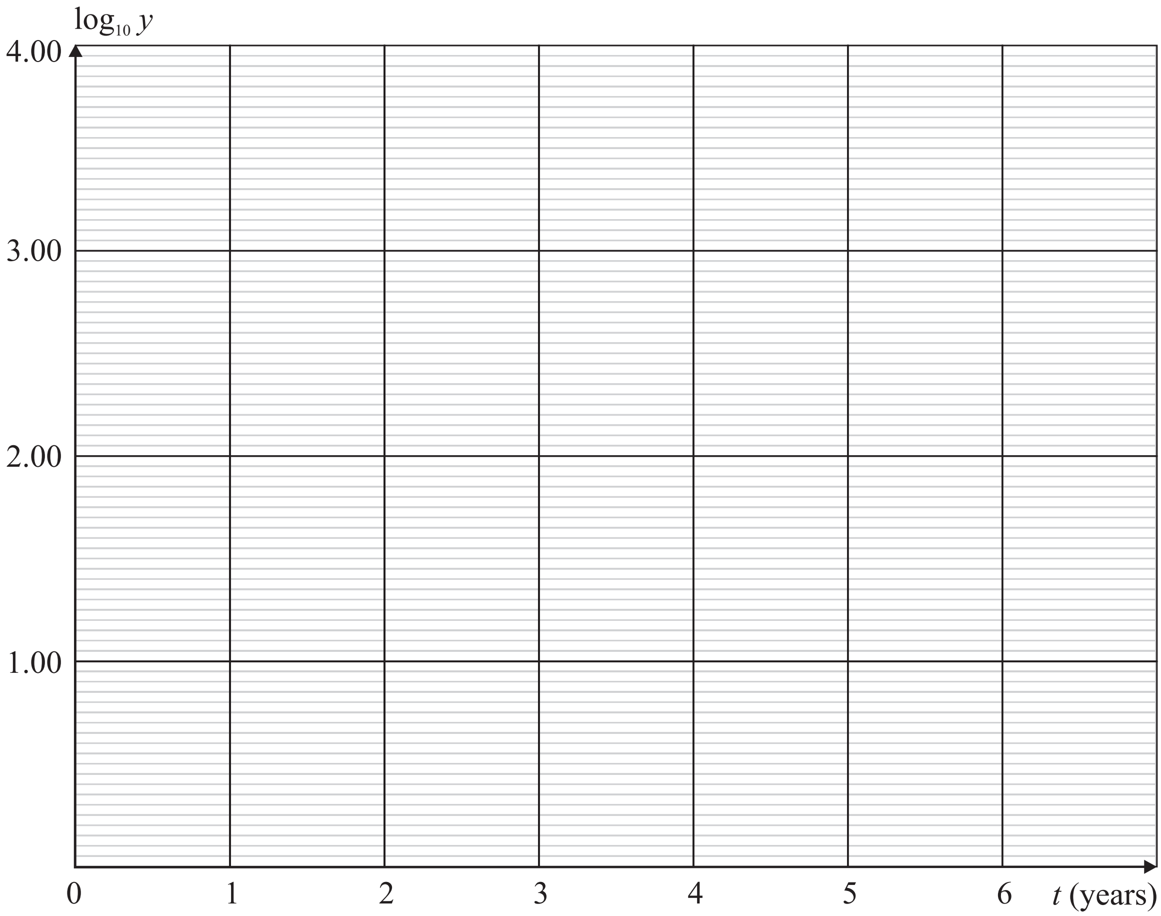
One often hears the expression 'an exponential increase'. The cost of health insurance in a certain country increased year on year according to the relationship given by $y = ab^t$, where y is the annual cost of insurance in euro, and t represents the number of years since 2006. This is an exponential relationship.

(a) Show that $y = ab^t$ may be written as $\log_{10} y = \log_{10} a + t \log_{10} b$.

(b) **(i)** The cost of health insurance in a certain country increased year on year after 2006 as shown in the table below. Fill in the last row of the table, writing each value to two decimal places.

t (Number of years after 2006)	1	2	3	4	5	6
y (Cost €)	500	610	720	890	1050	1540
$\log_{10} y$						

(ii) Plot $\log_{10} y$ against t on the graph below.



(c) Use the graph to find $\log_{10} a$ and $\log_{10} b$ and hence, the value of a, to the nearest whole number, and the value of b, to two decimal places.

(d) What was the cost of health insurance in 2006?

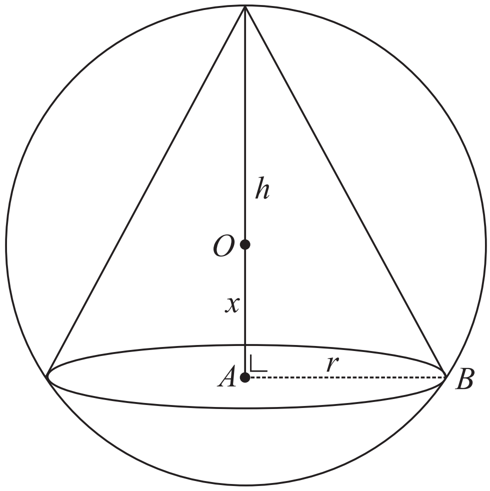
(e) Find the time t that will give a cost of €2200 to the nearest year.

Question 9 **(50 marks)**

A right-circular cone of base radius r and height h is inscribed in a sphere of centre O and radius 1.

(a) If $|OA| = x$, express:

(i) h in terms of x,



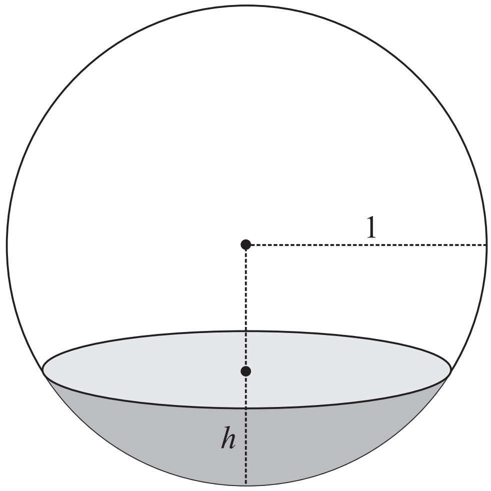
(ii) r in terms of x.

(b) Show that the volume of the cone is given by $V = \frac{1}{3}\pi(1 + x - x^2 - x^3)$.

(c) Find the maximum volume of the cone.

(d) For the maximum volume cone, show its semi-vertical angle is 35°, to the nearest degree.

(e) The volume of a section of sphere of radius 1 and height h is given by $\pi h^2 - \frac{1}{3}\pi h^3$.



If the cone with maximum volume is full of water and is leaking water so that the volume is decreasing at 0.2 cm^3 s^{-1}, find the rate at which the height of water in the sphere is increasing, to three decimal places, when the height of water is 0.3 cm.

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You may use this page for extra work.